ASSIGNMENT PROBLEMS

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Assignment problem is a special type of linear programming problem which deals with the allocation of the various resources to the various activities on one to one basis. It does it in such a way that

- the cost or time involved in the process is minimum and
- profit or sale is maximum.

Though there problems can be solved by simplex method or by transportation method but assignment model gives a simpler approach for these problems.

EXAMPLES

- A departmental head may have six people available for assignment and six jobs to assign. He may like to know which job be assigned to which person so that all these jobs can be completed in shortest possible time.
- Similarly in a marketing set up by making an estimate of sales performance for different territories, one could assign a particular salesman a particular territory with a view to maximise overall sales.

TABULAR PRESENTATION

Suppose there are n facilitates and n jobs, it is clear that in this case, there will be n assignments. Each facility or say worker can perform each job, one at a time. Then the problem is to find out which assignment should be made so that the profit is maximized or the cost or time is minimized.

Worker	Job of Work					
	1	2	3	4	jth	n
1.	Co_{11}	Co_{12}	Co_{13}	Co_{14}	.....	Co_{m}
2.	Co_{21}	Co_{22}	Co_{23}	Co_{24}	.....	Co_{2n}
3.	Co_{31}	Co_{32}	Co_{33}	Co_{34}	.....	Co_{3n}
i^{th}				Co_{ij}		
n^{th}	Co_{n1}	Co_{n2}	Co_{n3}	Co_{n4}	.....	Co_{nm}

In the table, Co_{ij} is defined as the cost when j^{th} job is assigned to i^{th} worker. It maybe noted here that this is a special case of transportation problem when the number of rows is equal to number of columns.

ASSUMPTIONS OF AN ASSIGNMENT PROBLEM

An assignment problem must satisfy the following assumptions:

1. The number of assignees and number of task are the same.
2. All persons are capable of doing all jobs but they vary in efficiency.
3. Each assignee is to be assigned to perform exactly one task.
4. There is a cost or profit associated with assignees performing different task.
5. The objective is to determine how all n assignment should be made to optimize the given pay offs which are expressed in terms of cost, time spent, distance, revenue earned, production obtained etc.

APPLICATION AREAS OF ASSIGNMNET PROBLEM

There exist numbers of areas where assignment problem can be used. In fact, whenever we have to make an assignment on one to one basis, assignment technique is used. For example,

- Assignments of different jobs to different workers
- Assignments of different machines to different workers
- Assignments of different salesmen to different sales centre/location
- Assignments of different products to different machines
- Assigning different rooms to different managers.
- Assigning teachers to classes

HUNGARIAN METHOD

The Hungarian mathematician D.Konig developed simpler and more efficient method of solving assignment problems which is known as Hungarian Method.

The method is based on the following principle:

If a constant is added to, or subtracted from, every element of a row and/or a column of the given cost matrix of an assignment problem, the resulting assignment problem has the same optimal solution as the original problem

STEPS TO FOLLOW

Consider the objective function of minimization type. Following steps are involved in solving this Assignment problem,

- **STEP 1 : ROW REDUCTION**

  Locate the smallest cost element in each row of the given cost table starting with the first row. Now, this smallest element is subtracted form each element of that row. So, we will be getting at least one zero in each row of this new table.

EXAMPLE

- A department has five employees with five jobs to be performed. The time (in hours) each men will take to perform each job is given in the effectiveness matrix.

Jobs/Employees	Sujay	Pawan	Prasad	Mohan	Hari
Job1	10	5	13	15	16
Job2	3	9	18	13	6
Job3	10	7	2	2	2
Job4	7	11	9	7	12
Job5	7	9	10	4	12

- How should the jobs be allocated, one per employee so as to minimise total number of manhours required to complete the job

Locate the smallest cost element in each row

Jobs/Employees	Sujay	Pawan	Prasad	Mohan	Hari
Job1	10	**5**	13	15	16
Job2	**3**	9	18	13	6
Job3	10	7	**2**	2	2
Job4	**7**	11	9	7	12
Job5	7	9	10	**4**	12

The smallest element is subtracted form each element of that row.

Jobs/Employees	Sujay	Pawan	Prasad	Mohan	Hari
Job1	5	**0**	8	10	11
Job2	**0**	6	15	10	3
Job3	8	5	**0**	0	0
Job4	**0**	4	2	0	5
Job5	3	5	6	**0**	8

STEP 2: COLUMN REDUCTION

Having constructed the table (as by step-1) take the columns of the table. Starting from first column locate the smallest cost element in each column.

Now subtract this smallest element from each element of that column. Having performed the step 1 and step 2, we will be getting at least one zero in each column in the reduced cost table.

The smallest element in each column is zero. So the same matrix will be obtained after column reduction.

Jobs/Employees	Sujay	Pawan	Prasad	Mohan	Hari
Job1	5	**0**	8	10	11
Job2	**0**	6	15	10	3
Job3	8	5	**0**	0	0
Job4	**0**	4	2	0	5
Job5	3	5	6	**0**	8

STEP 3 : MAKING ASSIGNMENT

Now, the assignments are made for the reduced table in following manner.

- Rows are examined successively, until the row with exactly single (one) zero is found. Assignment is made to this single zero by putting square □ around it and in the corresponding column, all other zeros are crossed out (x) because these will not be used to make any other assignment in this column. Step is conducted for each row.
- Above step is now performed on the columns. Columns are examined successively till a column with exactly one zero is found. Now , assignment is made to this single zero by putting the square around it and at the same time, all other zeros in the corresponding rows are crossed out (x) step is conducted for each column.
- Above two steps are repeated till all the zeros are either marked or crossed out

Jobs/Employees	Sujay	Pawan	Prasad	Mohan	Hari
Job1	5	**0**	8	10	11
Job2	**0**	6	15	10	3
Job3	8	5	**0**	~~0~~	~~0~~
Job4	~~0~~	4	2	~~0~~	5
Job5	3	5	6	**0**	8

CHECK FOR OPTIMAL SOLUTION

- Now, if the number of marked zeros or the assignments made are equal to number of rows or columns, optimum solution has been achieved. There will be exactly single assignment in each row or each column.
- But if we find certain row or column without any assignment, in this case, we will go to next step.

STEP 4

Draw the minimum number of lines (horizontal and vertical) necessary to cover all zeros in the matrix obtained in step 3, Following procedure is adopted:

(i)Tick mark (√) all rows that do not have any assignment

(ii) Now tick mark(√) all these columns that have zero in the tick marked rows.

(iii) Now tick mark all the rows that are not already marked and that have assignment in the marked columns.

(iv) All the steps i.e. (4(i), 4(ii), 4(iii) are repeated until no more rows or columns can be marked.

(v) Now draw straight lines which pass through all the unmarked rows and marked columns. It can also be noticed that in an n x n matrix, always less than ‘n’ lines will cover all the zeros if there is no solution among them.

Jobs/Employees	Sujay	Pawan	Prasad	Mohan	Hari
Job1	5	0	8	10	11
Job2	0	6	15	10	3
Job3	8	5	0	0	0
Job4	0	4	2	0	5
Job5	3	5	6	0	8

- In step 4, if the number of lines drawn are equal to n or the number of rows, then it is the optimum solution if not, then go to next step.
- Select the smallest element among all the uncovered elements. Now, this element is subtracted from all the uncovered elements and added to the element which lies at the intersection of two lines. This is the matrix for fresh assignments.
- Repeat the procedure from step (3) until the number of assignments becomes equal to the number of rows or number of columns.

Jobs/Employees	Sujay	Pawan	Prasad	Mohan	Hari
Job1	5	**0**	8	10	11
Job2	**0**	6	15	10	3
Job3	8	5	**0**	0	0
Job4	0	4	2	0	5
Job5	3	5	6	**0**	8

Jobs/Employees	Sujay	Pawan	Prasad	Mohan	Hari
Job1	7	**0**	8	12	11
Job2	**0**	4	13	10	1
Job3	10	5	**0**	2	0
Job4	**0**	2	0	0	3
Job5	3	3	4	**0**	6

Jobs/Employees	Sujay	Pawan	Prasad	Mohan	Hari
Job1	7	**0**	8	12	11
Job2	**0**	4	13	10	1
Job3	10	5	~~0~~	2	0
Job4	~~0~~	2	0	~~0~~	3
Job5	3	3	4	**0**	6

Solution of job allocation is as below. Total time: 23 man hours

Job	Job1	Job2	Job3	Job4	Job5
Employee	Pawan	Sujay	Hari	Prasad	Mohan
Time	5	3	2	9	4

Unbalanced Assignment Problem

- It is an assignment problem where the number of persons is not equal to the number of jobs.
- If the number of persons is less than the number of jobs then we introduce one or more dummy persons (rows) with zero values to make the assignment problem balanced.
- Likewise, if the number of jobs is less than the number of persons then we introduce one or more dummy jobs (columns) with zero values to make the assignment problem balanced

Jobs				
Persons	1	2	3	4
A	20	25	22	28
B	15	18	23	17
C	19	17	21	24

Since the number of persons is less than the number of jobs, we introduce a dummy person (D) with zero values. The revised assignment problem is given below:

Jobs				
Persons	1	2	3	4
A	20	25	22	28
B	15	18	23	17
C	19	17	21	24
D (dummy)	0	0	0	0

Maximisation case in Assignment Problem

- Some assignment problems entail maximizing the profit, effectiveness, or layoff of an assignment of persons to tasks or of jobs to machines.
- The conversion is accomplished by subtracting all the elements of the given effectiveness matrix from the highest element.
- It turns out that minimizing opportunity loss produces the same assignment solution as the original maximization problem.

- Five different machines can do any of the five required jobs, with different profits resulting from each assignment as given below:

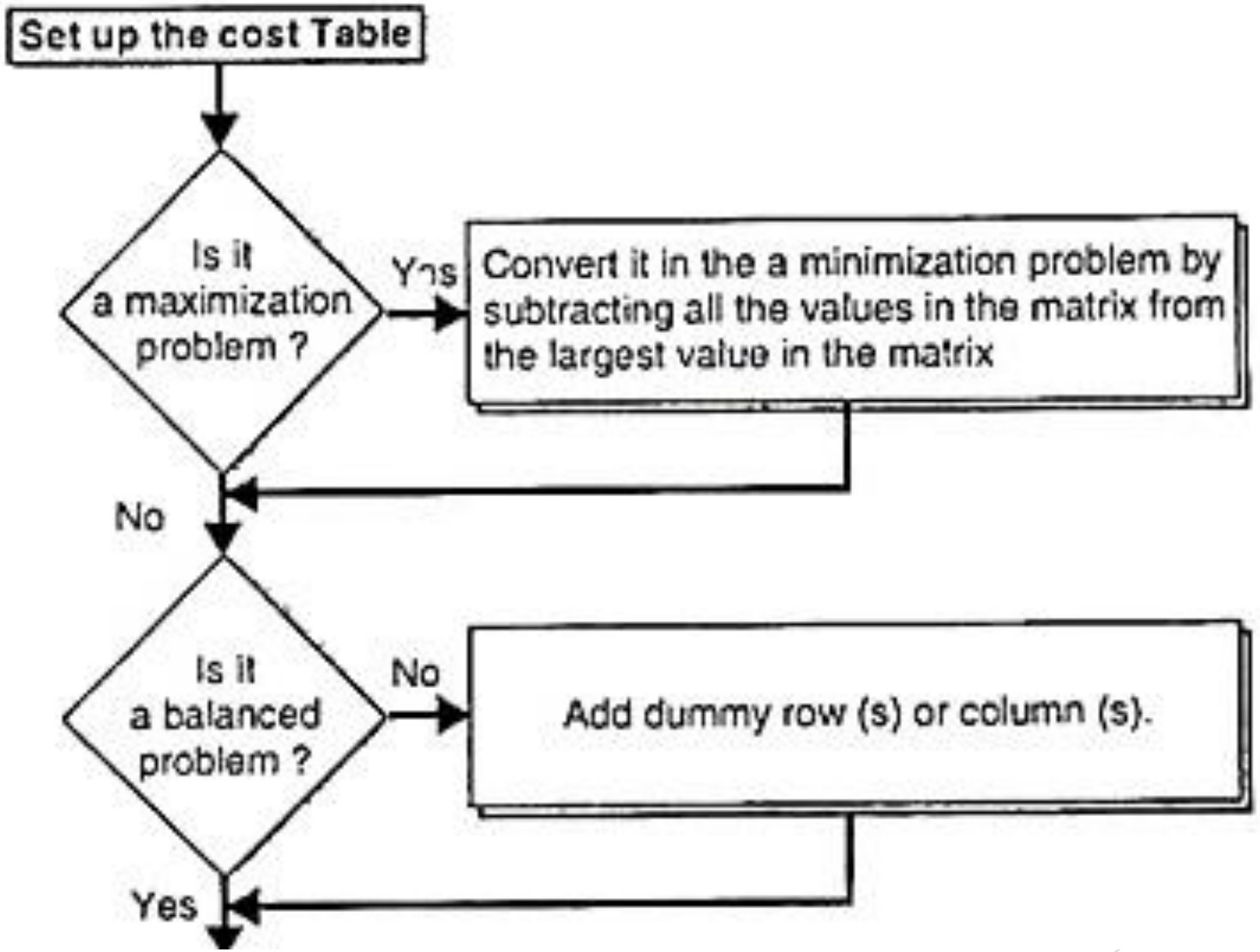
Machines					
Jobs	**A**	**B**	**C**	**D**	**E**
1	30	37	40	28	40
2	40	24	27	21	36
3	40	32	33	30	35
4	25	38	40	36	36
5	29	62	41	34	39

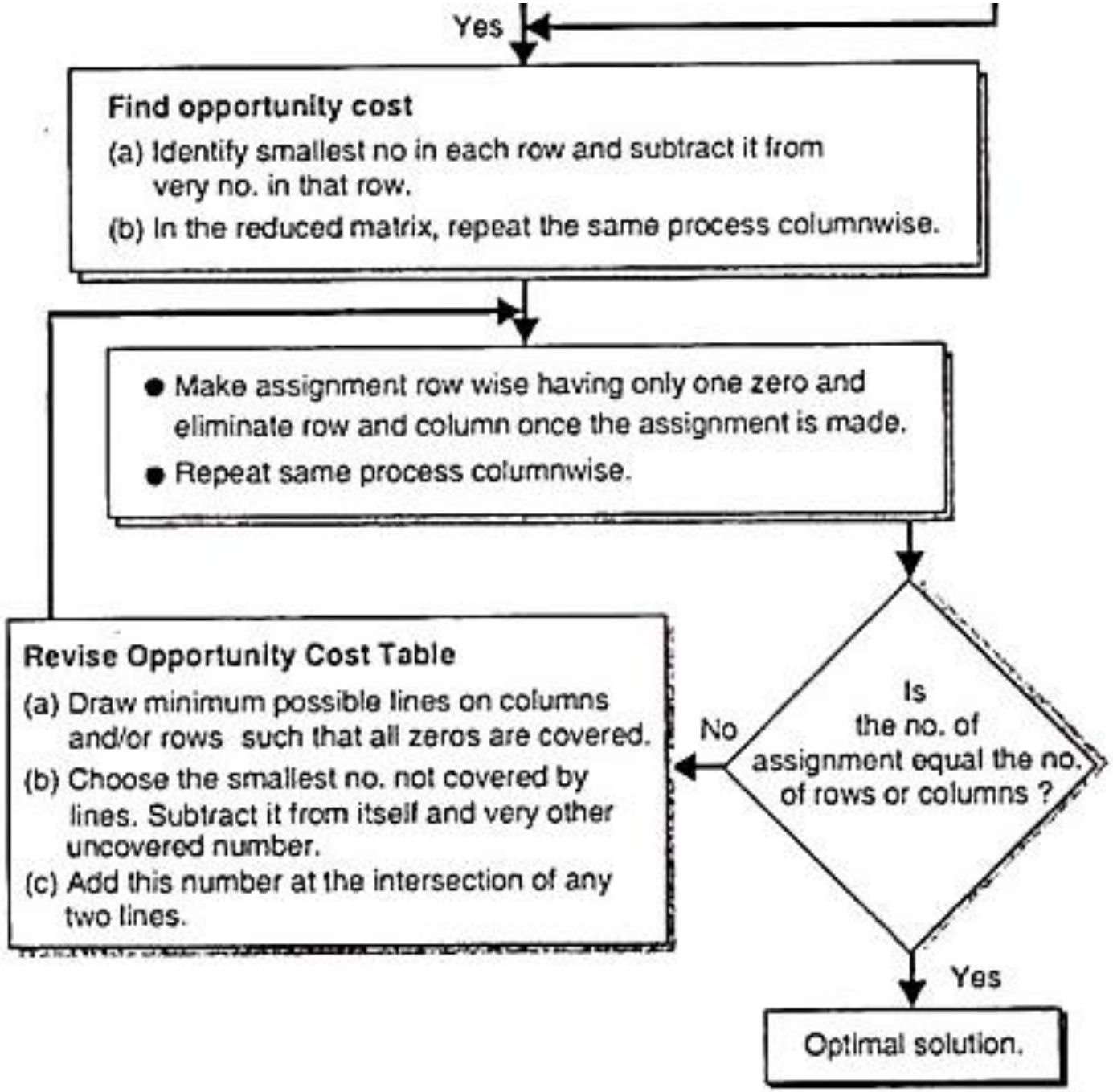
Find out the maximum profit possible through optimum assignment.

- Here, the highest element is **62. So we subtract each value from 62.**

Machines					
Jobs	**A**	**B**	**C**	**D**	**E**
1	32	25	22	34	22
2	22	38	35	41	26
3	22	30	29	32	27
4	37	24	22	26	26
5	33	0	21	28	23

Flow chart of steps in the Hungarian Mehod




Yes
Find opportunity cost
(a) Identify smallest no in each row and subtract it from very no. in that row.
(b) In the reduced matrix, repeat the same process columnwise.
● Make assignment row wise having only one zero and eliminate row and column once the assignment is made.
● Repeat same process columnwise.
Is the no. of assignment equal the no. of rows or columns ?
No
Revise Opportunity Cost Table
(a) Draw minimum possible lines on columns and/or rows such that all zeros are covered.
(b) Choose the smallest no. not covered by lines. Subtract it from itself and very other uncovered number.
(c) Add this number at the intersection of any two lines.
Yes
Optimal solution.

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Thank
you