Introduction to Probability

Gagan Deep
Department of Mathematics
HMV, Jalandhar

Random Experiment

A experiment which is performed under similar conditions gives different outcomes

Sample Space (S): Collection of all possible outcomes of random Experiment

Example 1: Tossing of a die $S = (H, T)$

Example 2: Tossing of a coin $S = \{1, 2, \ldots, 6\}$

Deterministic Experiment

A Experiment which is performed under similar conditions gives the same result

Example: A body moving with initial velocity U, acceleration a will have final velocity after time t

$$V = U + at$$

Probability

If a random experiment results in *n* exhaustive mutually exclusive and equally likely outcome, out of which m are favorable to the occurrence of event *E*, then the probability 'p' of occurrence of *E* is given by

$$p = P(E) = \frac{\text{Number of favourable cases}}{\text{Total number of exhaustive cases}} = \frac{m}{n}$$

Limitations:

- Outcomes are not equally likely
- If the number of outcomes of the random experiment are infinite

Statistical Definition of Probability

If an experiment is performed repeatedly under essentially homogeneous and identical conditions, then the limiting value of the ratio of the number of times the events occurs to the number of trials, as the number of trials becomes indefinitely large, is called the probability of happening of the event.

M= Number of times event A occurs in N trials

$$P(E) = \lim_{n \to \infty} \frac{M}{N}$$

Limitations: Experiment repeated may not be identical

The limit may not attain unique value, however large N may be

Random Variable

A random variable is a real value with domain as a sample space of the random experiment

Example 1: In experiment of tossing a coin with sample space $S = \{H, T\}$

The random variable X can be defined as

$X(H) = 1$

$X(T) = 0$

Example2 : Consider an experiment, tossing ow two coins $S = \{HH, HT, TH, TT\}$

X = number of heads

$X(HH) = 2, \; X(HT) = 1, \; X(TH) = 1, \; X(TT) = 0$

$$P[X \leq 1] P[w : w \in S \text{ and } X(W) \leq 1] = P[HT, HT, TH] = \frac{3}{4}$$

Types of Random Variables

Discrete Random Variable: Finite or countably infinite values

Example

X=outcomes of coined tossed

X=Number of accidents can occurs in a city

Continuous Random Variable: Taking all values in some intervals of real lines

X=Age of a living being

Probability Mass Function

X: Discrete random variable taking values $x_1, \ldots, x_n, \ldots$

Probability mass function satisfying following properties

$$P(x_i) \geq 0 \quad for\ all\ \ x_i$$

$$\sum_i P(x_i) = 1$$

$$P(h(x)) = \sum_{x_i : h(x_i) = x} P(x_i)$$

Probability Density Function

X- continuous Random Variable

Pdf $f(x)$ is a function satisfying following properties

$$f(x) \geq 0$$

$$\int_{-\infty}^{\infty} f(x)dx = 1$$

$$P(a < X < b) = \int_{a}^{b} f(x)dx$$

Cumulative Distribution Function

$$F(x) = P(X \leq x)$$

$$= \begin{cases} \sum_{x_i \leq x} p(x_i) & if \;\; X \;\; is \; discrete \\ \int_{-\infty}^{x} f(x)dx & if \;\; X \;\; is \; continous \end{cases}$$

THANKS