Classical to Quantum Mechanics

Photoelectric effect

- For a particular metal and a given color of light, say blue, it is found that the electrons come out with a well-defined speed, and that the number of electrons that come out depends on the intensity of the light.
- If the intensity of light is increased, more electrons come out, but each electron has the same speed, independent of the intensity of the light.
- If the color of light is changed to red, the electron speed is slower, and if the color is made redder and redder, the electrons' speed is slower and slower.
- For red enough light, electrons cease to come out of the metal.

Failure of classical theory to explain Photoelectric effect

Using the classical Maxwell wave theory of light,

- **the more intense the incident light the greater the energy** with which the electrons should be ejected from the metal. That is, the average energy carried by an ejected (photoelectric) electron should increase with the intensity of the incident light.

Hence it is expected : **lag time** between exposure of the metal and emission of electron.

Therefore, classical theory could not explain the characteristics of photoelectric effect:

- Instantaneous emission of electrons
- Existence of threshold frequency
- Dependence of kinetic of the emitted electron on the frequency of light.

Einstein explanation of Photoelectric effect

Einstein (1905) successfully resolved this paradox by employing Planck's idea of quantization of energy and proposed that the incident light consisted of individual quanta, called photons, that interacted with the electrons in the metal like discrete particles, rather than as continuous waves.

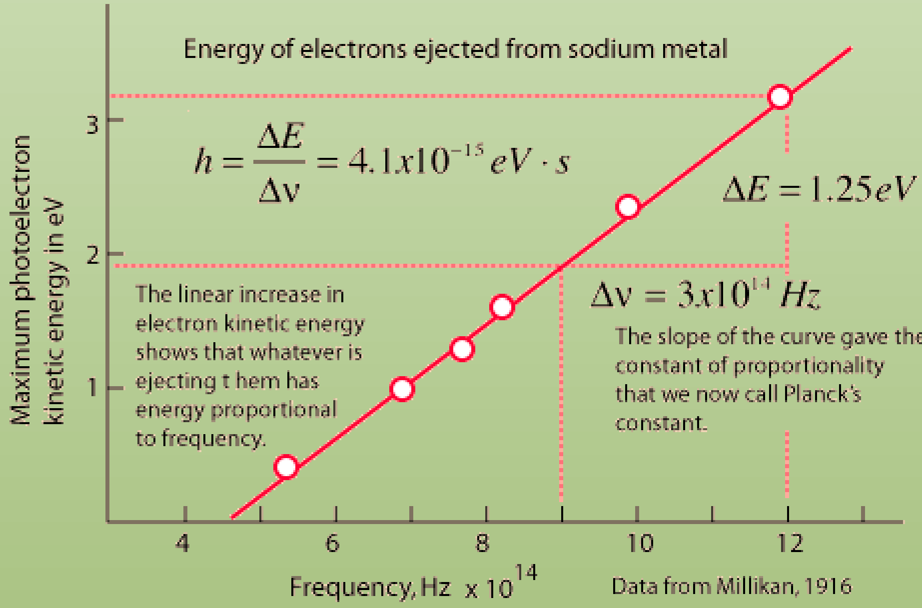
$h\nu = K.E + W$

where ν is frequency of radiation, K.E. is kinetic energy of emitted electron, W is work potential

$W = h\nu_0$ (ν_0 is threshold frequency)

$K.E. = h\nu - h\nu_0$

$K.E. = h(\nu - \nu_0)$



Though most commonly observed phenomena with light can be explained by waves. But the photoelectric effect suggested a **particle nature for light**.

Wave-Particle Duality: Light

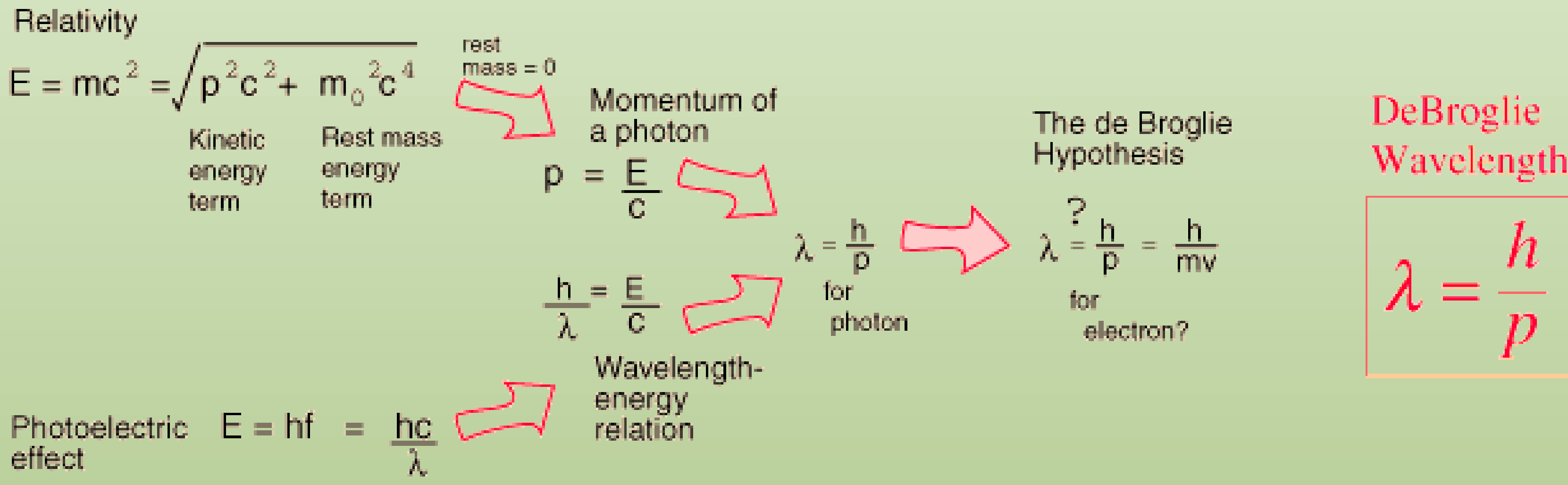
Does light consist of particles or waves? When one focuses upon the different types of phenomena observed with light, a strong case can be built for a wave picture:

Phenomenon	Can be explained in terms of waves.	Can be explained in terms of particles.
Reflection	∿∿ ✓	•→ ✓
Refraction	∿∿ ✓	•→ ✓
Interference	∿∿ ✓	•→ ⊗
Diffraction	∿∿ ✓	•→ ⊗
Polarization	∿∿ ✓	•→ ⊗
Photoelectric effect	∿∿ ⊗	•→ ✓
Compton scattering	∿∿ ⊗	•→ ✓

Most commonly observed phenomena with light can be explained by waves. But the photoelectric effect and the Compton scatering suggested a particle nature for light. Then electrons too were found to exhibit dual natures.

Wave Nature of Electron

As a young student at the University of Paris, Louis DeBroglie had been impacted by relativity and the photoelectric effect, both of which had been introduced in his lifetime. The photoelectric effect pointed to the particle properties of light, which had been considered to be a wave phenomenon. He wondered if electons and other "particles" might exhibit wave properties. The application of these two new ideas to light pointed to an interesting possibility:



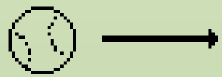
Confirmation of the DeBroglie hypothesis came in the **Davisson- Germer experiment** which showed interference patterns – in agreement with **DeBroglie wavelength** – for the scattering of electrons on nickel crystals.

DeBroglie Wavelengths

DeBroglie Wavelength

$$\lambda = \frac{h}{p}$$

Does this relationship apply to all particles? Consider a pitched baseball:



$v = 40$ m/s $= 90$ mi/hr

m=0.15 kg

$$\lambda = \frac{h}{mv} = \frac{6.626 \times 10^{-34}\ \text{J s}}{(0.15\ \text{kg})(40\ \text{m/s})} = 1.1 \times 10^{-34}\ \text{m}$$

10^{-10}m Atomic diameter
10^{-14}m Nuclear Diameter

For an electron accelerated through 100 Volts: $v = 5.9 \times 10^{6}$ m/s

$$\lambda = \frac{6.626 \times 10^{-34}\ \text{Js}}{(9.11 \times 10^{-31}\ \text{kg})(5.9 \times 10^{6}\ \text{m/s})} = 1.2 \times 10^{-10} = 0.12\ \text{nm}$$

This is on the order of atomic dimensions and is much shorter than the shortest visible light wavelength of about 390 nm.

- **The de Broglie wavelength λ for macroscopic particles are negligibly small**

- **This effect is extremely important for light particles, like electrons.**

Wave or Particle (Size ??)

- Objects that are large in the absolute sense have the property that the wavelengths associated with them are completely negligible compared to their size. Therefore, large particles only manifest their particle nature, they never manifest their wave nature.

Quantization of angular momentum

Bohr’s postulate

Bohr Atomic Model :
Angular Momentum Quantization

- Bohr proposed a quantum restriction on classical model i.e. the angular momentum of the revolving electron is an integral multiple of basic unit $(h/2\pi)$

$$\mathbf{mvr = n\ (h/2\pi)}$$

❑Combining the energy of the classical electron orbit with the quantization of angular momentum, the Bohr approach yields expressions for the electron orbit radii and energies:

$$\frac{mv^2}{2} = \frac{(mvr)^2}{2mr^2} = \frac{n^2h^2}{8\pi^2 mr^2} = \frac{Ze^2}{8\pi\varepsilon_0 r}$$

kinetic energy of electron	expressed in terms of angular momentum	use quantization of angular momentum	set equal to total energy of classical orbit

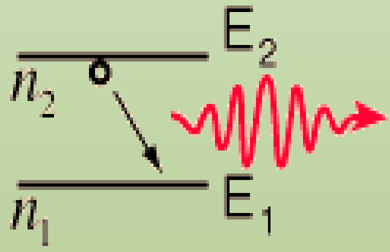
This is for hydrogenic atoms; the use of the atomic number Z is appropriate only if there is only one electron.

$$E = -\frac{Z^2 me^4}{8n^2h^2\varepsilon_0^2} = -\frac{13.6Z^2}{n^2}\ \text{eV}$$

$$r = \frac{n^2h^2\varepsilon_0}{Z\pi me^2} = \frac{n^2 a_0}{Z}$$

a_0 = 0.529 Å = Bohr radius

The Bohr model for an electron transition in hydrogen between quantized energy levels with different quantum numbers n yields a photon by emission, with quantum energy



A downward transition involves emission of a photon of energy:

$$E_{photon} = h\upsilon = E_2 - E_1$$

Given the expression for the energies of the hydrogen electron states:

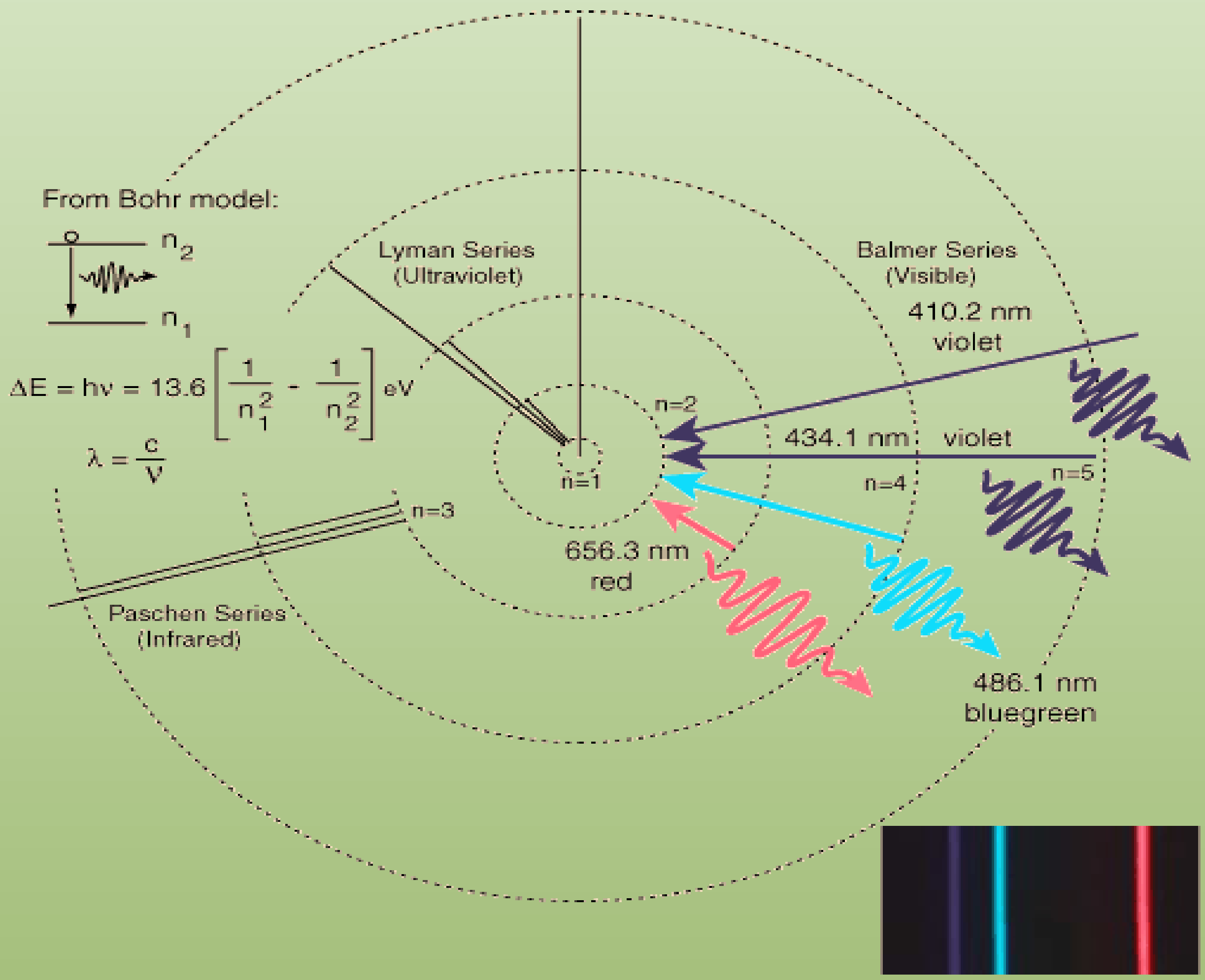
$$h\nu = \frac{me^4}{8\varepsilon_0^2 h^2}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right] = 13.6\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right] \text{eV}$$

This is often expressed in terms of the inverse wavelength or "wave number" as follows:

Ritz Equation

$$\frac{1}{\lambda} = R_H \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right] \quad \text{where}$$

$$R_H = \frac{me^4}{8\varepsilon_0{}^2 ch^3} \qquad R_H = 1.097 \ 10^{-7} \ \text{m}^{-1}$$


From Bohr model:
n_2
n_1
$\Delta E = h\nu = 13.6 \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right] eV$
$\lambda = \frac{c}{\nu}$
Lyman Series
(Ultraviolet)
Balmer Series
(Visible)
410.2 nm
violet
434.1 nm violet
n=2
n=1
n=3
n=4
n=5
656.3 nm
red
486.1 nm
bluegreen
Paschen Series
(Infrared)

Spectral series Shown by Hydrogen atom:

Lyman series (ultraviolet region) : $n_1 = 1$, $n_2 = 2, 3, 4, \ldots..$

Balmer Series (visible region) : $n_1 = 2$, $n_2 = 3, 4, 5 \ldots..$

Paschen Series (infrared region): $n_1 = 3$, $n_2 = 4, 5, 6 \ldots..$

Brackett Series (infrared region): $n_1 = 4$, $n_2 = 5, 6, 7 \ldots..$

Pfund series (far infrared region): $n_1 = 5$, $n_2 = 6, 7, 8 \ldots..$

Thank You