PAPER-A: CONDENSED MATTER PHYSICS (B.Sc. Semester-V)

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CONTENTS

- Point Group Symmetry
- 5-Fold Symmetry

Point group symmetry

- Inorganic crystals usually have perfect shape which reflects their internal symmetry
- Point groups are originally used to describe the symmetry of crystal.
- Point group symmetry does not consider translation.
- Included symmetry elements are rotation, mirror plane, center of symmetry, rotary inversion.

Point group symmetry diagrams

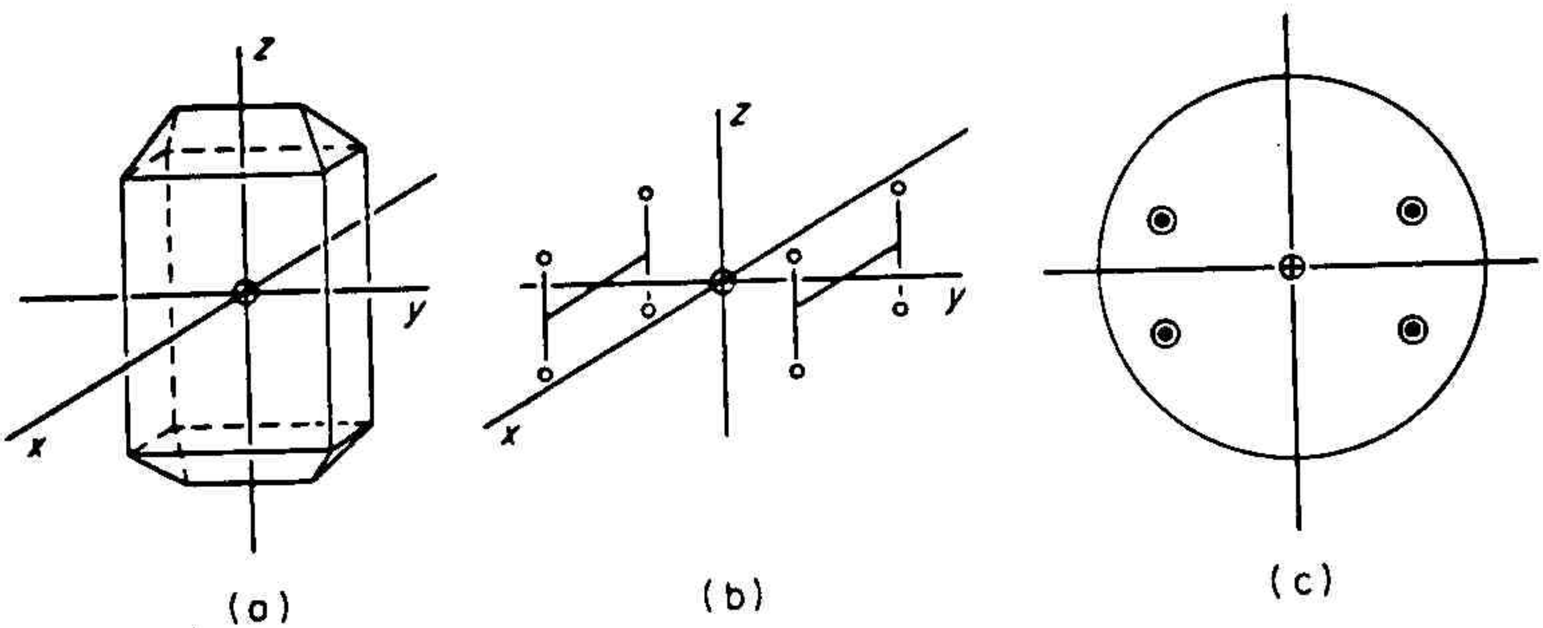


Figure 3.18. (*a*) Crystal with symmetry *mmm*. (*b*) Set of points related by symmetry *mmm*. (*c*) Plane representation of symmetry *mmm*.

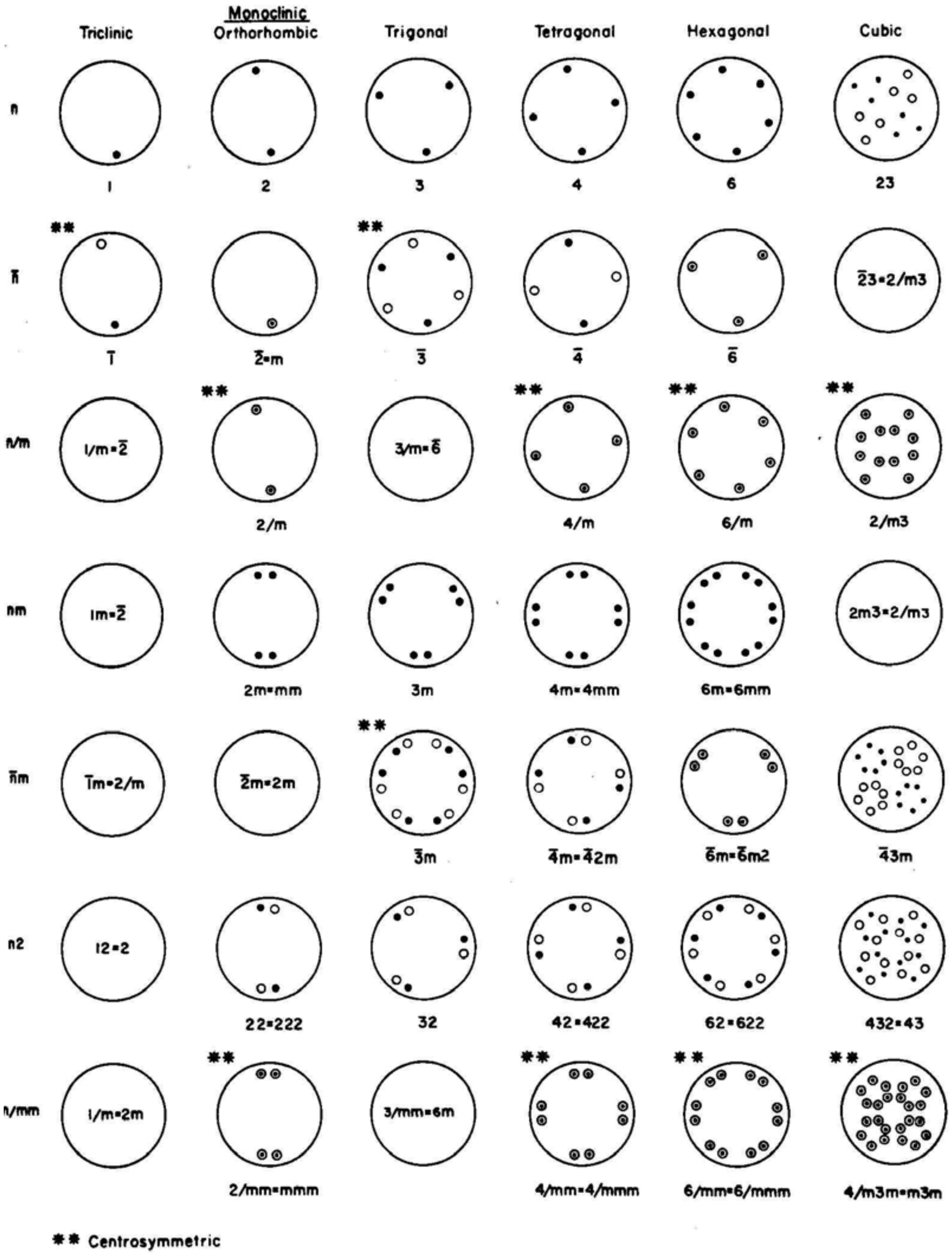
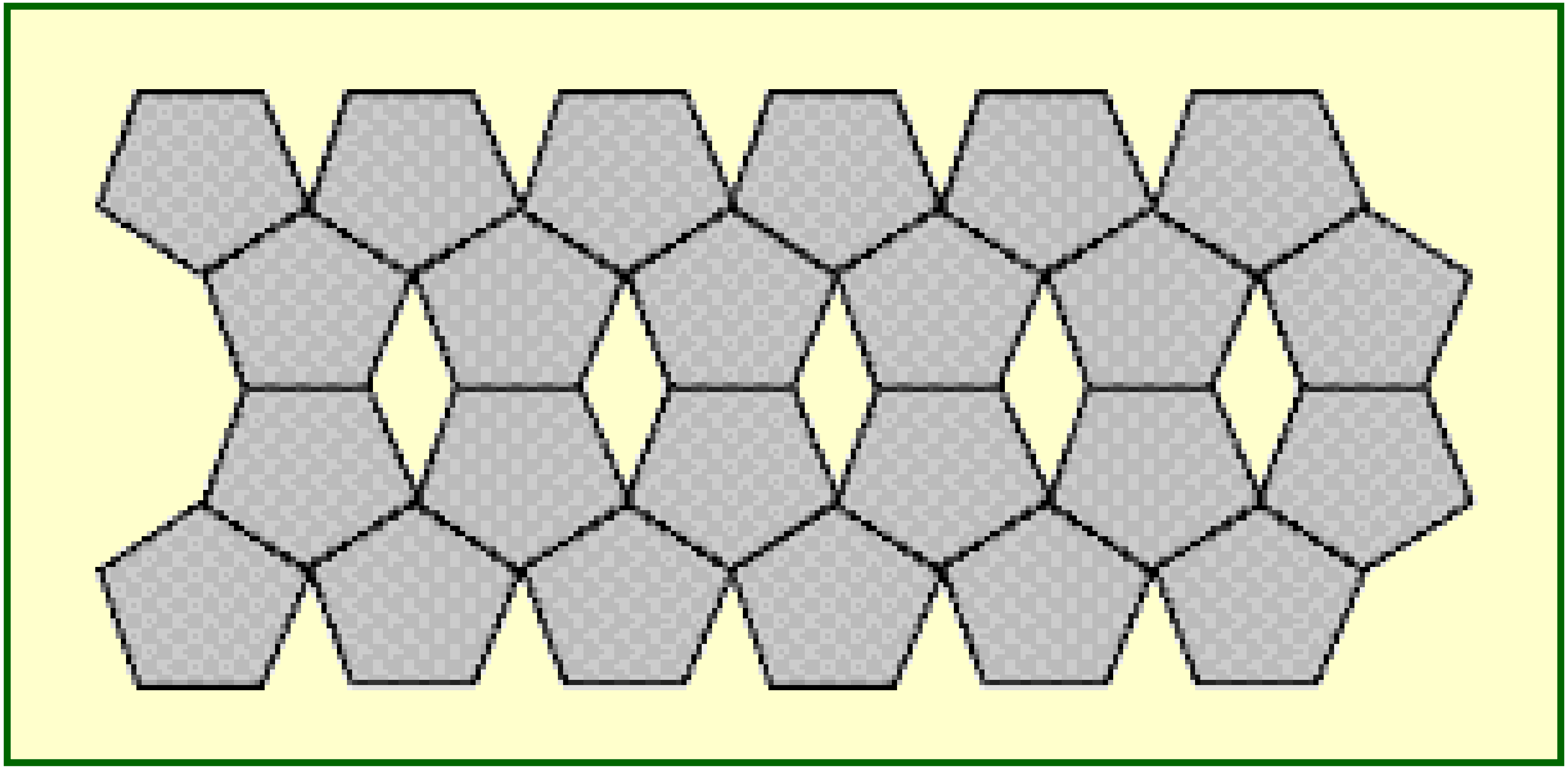


Figure 3.19. Plane representations of the 32 point groups.

There are a total of 32 point groups

5-Fold Symmetry



This type of symmetry is not allowed because it can not be combined with translational periodicity!

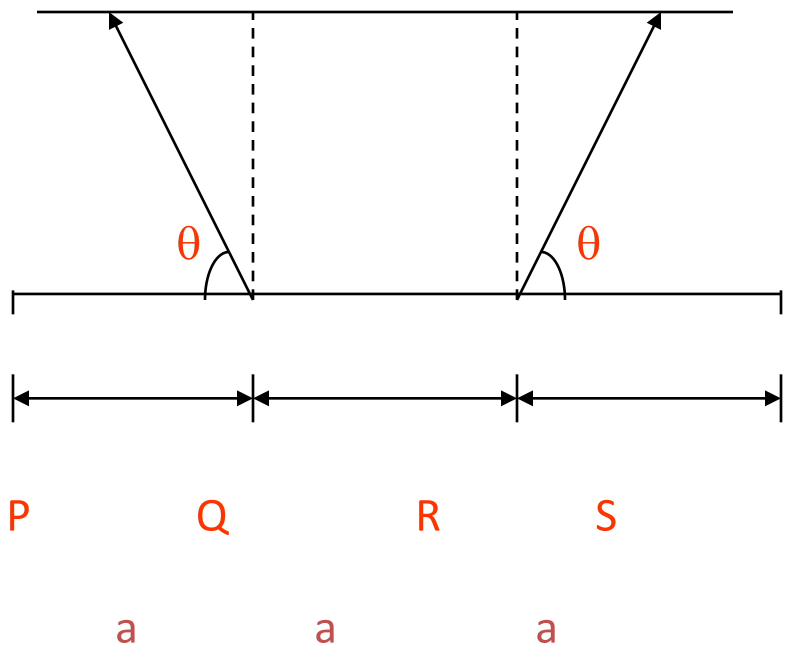
ABSENCE OF 5 FOLD SYMMETRY

- We have seen earlier that the crystalline solids show only 1,2,3,4 and 6-fold axes of symmetry and not 5-fold axis of symmetry or symmetry axis higher than 6.

- The reason is that, a crystal is a one in which the atoms or molecules are internally arranged in a very regular and periodic fashion in a three dimensional pattern, and identical repetition of an unit cell can take place only when we consider 1,2,3,4 and 6-fold axes.

MATHEMATICAL VERIFICATION

Let us consider a lattice P Q R S as shown in figure



MATHEMATICAL VERIFICATION

- Let this lattice has n-fold axis of symmetry and the lattice parameter be equal to ‘a’.
- Let us rotate the vectors Q P and R S through an angle $\theta = \frac{360^o}{n}$ in the clockwise and anti clockwise directions respectively.
- After rotation the ends of the vectors be at x and y.
- Since the lattice PQRS has n-fold axis of symmetry, the points x and y should be the lattice points.

MATHEMATICAL VERIFICATION

- Further the line xy should be parallel to the line PQRS. Therefore the distance xy must equal to some integral multiple of the lattice parameter ‘a’ say, m a.

i.e., $xy = a + 2a\cos\theta = ma$ (1)

Here, $m = 0, \pm 1, \pm 2, \pm 3, \ldots\ldots\ldots\ldots\ldots\ldots$

From equation (1),

$$2a\cos\theta = m\,a - a$$

MATHEMATICAL VERIFICATION

i.e., $2a \cos\theta = a(m - 1)$

(or) $\cos\theta = \dfrac{m-1}{2} = \dfrac{N}{2}$ (2)

Here,

$N = 0, \pm 1, \pm 2, \pm 3, \ldots$..

since (m-1) is also an integer, say N.

We can determine the values of θ which are allowed in a lattice by solving the equation (2) for all values of N.

MATHEMATICAL VERIFICATION

- For example, if $N = 0$, $\cos\theta = 0$ i.e., $\theta = 90^o$

  $\therefore$ $n = 4$.
- In a similar way, we can get four more rotation axes in a lattice, i.e., $n = 1$, $n = 2$, $n = 3$, and $n = 6$.
- Since the allowed values of $\cos\theta$ have the limits -1 to $+1$, the solutions of the equation (2) are not possible for $N > 2$.
- Therefore only 1, 2, 3, 4 and 6 fold symmetry axes can exist in a lattice.

ROTATION AXES ALLOWED IN A LATTICE

N	N/2	cos θ	θ(degrees)	n= $\left(\frac{360^0}{\theta}\right)$
-2	-1	-1	180	2
-1	-1/2	-1/2	120	3
0	0	0	90	4
+1	+1/2	+1/2	60	6
+2	+1	+1	360 (or) 0	1

REFERENCES

- Introduction To Solid State Physics- Kittle
- Elementary Solid state Physics- Omar
- Solid state Physics- S. O. Pillai

THANK YOU !